VECTORS

Quantities can be classified based on derivation or direction. Based on derivation, quantities can be either fundamental or derived. Based on direction, they could be *scalar* or *vector* quantities.

Scalar quantities are those quantities that have only magnitude but no direction. Examples are volume, area, length, time, density, speed, temperature, energy, etc.

Apart from scalar quantities, many other quantities have both magnitude and direction. Such quantities are not completely described unless their magnitude and directions are specified. These are known as vector quantities. Examples are displacement, velocity, acceleration, work, force, weight, momentum, electric field, etc.

Vector Representation

Since a vector quantity has both magnitude and direction, we represent it by a *line* (magnitude) which

has an *arrowhead* (indicating the direction of the given vector). The length of the line is drawn to represent the magnitude of the vector; the arrowhead is always drawn so that it points in the direction of the vector.

We represent a displacement of 300km to the East by a straight line AB, 3cm long, which points towards the east from our chosen origin. The scale used for the representation is 1cm to 100km.

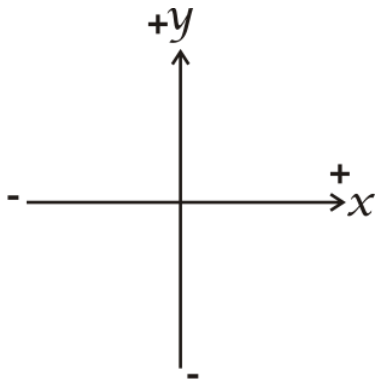


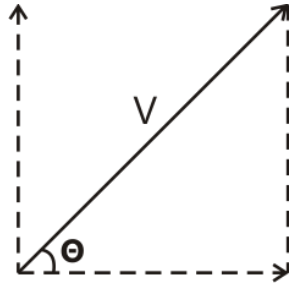
Addition of Vectors

Scalar quantities can be added up by simple algebraic process. For example, to add a sped of $30kmhr^{-1}$ to a speed of $10kmhr^{-1}$, we merely sum up the two magnitudes to obtain a speed of $40kmhr^{-1}$.

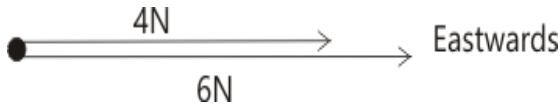
On the other hand, vector quantities can't be added up by simple algebraic process, as they have direction (angle and direction sign). They are usually added up using geometric processes. Below is the Cartesian plane. Notice how signs are used to imply

direction and how vectors' direction could form a triangle and an inclination angle.

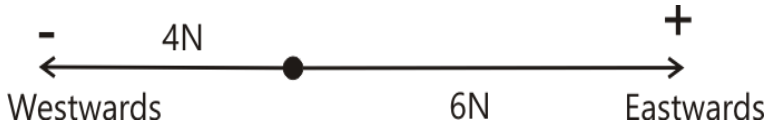




Linear Case: Suppose we have two forces acting on an object 4N and 6N, and they act on the body in the same direction (say East); the resultant force (or vector) would be $4N + 6N = 10N$ Eastwards as they act in the same direction.

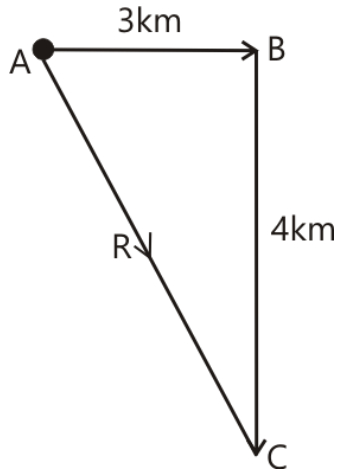


If these forces act in opposite direction say east and west, then the sign of cardinality or direction would be use.



The resultant vector would be: $6N - 4N = 2N$ Eastwards

Right-angle Case: When two vectors act in directions that are perpendicular to each other, we can simplify by using Pythagoras theorem. For instance, a man starts from A and walks 3km eastwards and then 4km southwards, find the resultant vector (displacement).



To find R, we use Pythagoras theorem: $AC^2 = AB^2 + BC^2$

AB = 3km; BC= -4km (– represents direction)

$AC^2 = 3^2 + (-4)^2 = 9 + 16 = 25$

$AC = \sqrt{25} = 5km$

∴ the resultant vector (displacement) R is 5km.

Also the angle of inclination or rather the direction of the resultant vector can be gotten by using trigonometric ratios.

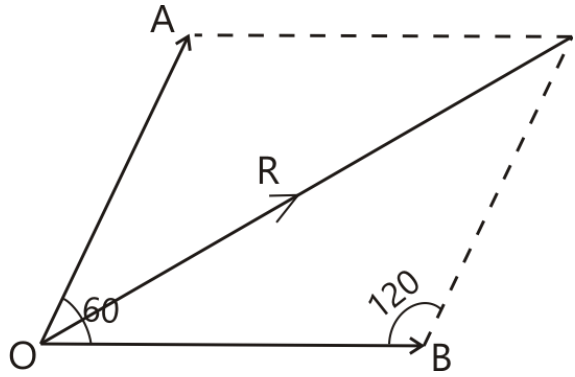
$$tan\theta = \frac{opp}{adj} = \frac{4}{3} = 1.333\ (3dp)$$

$$\theta = \tan^{-1} 1.333 = 53.13^o\ (2dp)$$

Thus the *resultant vector R* is 5km 53.13° SE.

Obtuse angle Case: The resultant of two vectors inclined to each other at an angle other than 90° can be simplified using *parallelogram method*. That is the vectors can be represented by the sides of parallelogram and the resultant represented by the

diagonal that originates from the point of intersection of the two vectors. For example: two forces A and B act on an object at 60^{o} to each other. This could be represented as below:



R is the resultant vector of OA and OB. OA and OB are the vectors acting on the object at O.

We use *Cosine Rule* of mathematics to find R in relation to A and B.

$$R^2 = OA^2 + OB^2 + 2ABcos\theta$$

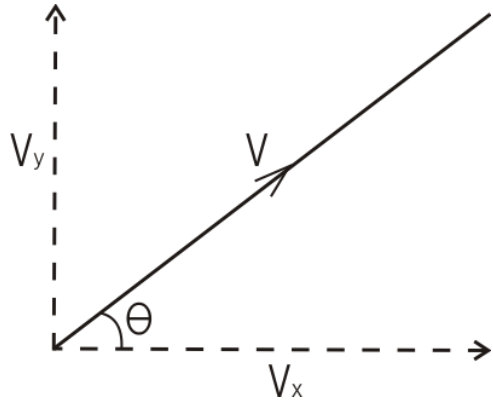
The *Parallelogram law of vectors* states that if two vectors are represented in magnitude and direction by the adjacent sides of a parallelogram, the diagonal of the parallelogram drawn from the point of intersection

of the vectors represents the resultant vector in magnitude and direction.

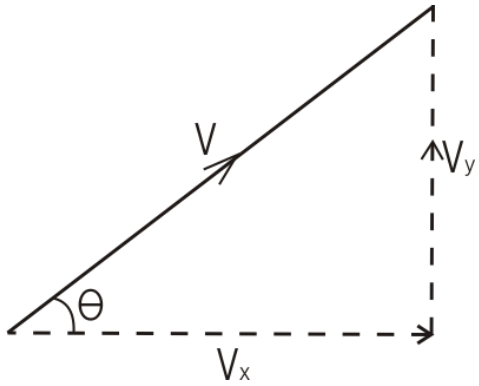
For finding the angle (or direction) of inclination of the resultant vector, we use *Sine rule* which is

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

The Resultant of more than two vectors: To find the resultant of more than two vectors, we first resolve each vector in two perpendicular vectors; add all the horizontal components, x and all the vertical components, y.



Resolution of vector is the process of finding the two perpendicular vectors that would result from the vector to be resolved. In other words we can say that resolution of vectors is the finding of the x and y values of a vector. To find this value we use simple trigonometric ratios:



$$sin\theta = \frac{opposite}{hypotenuse} = \frac{V_y}{V}$$

$$V_y = Vsin\theta$$

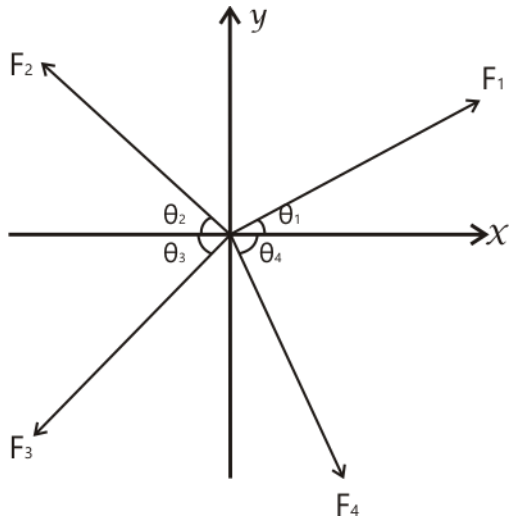
$$cos\theta = \frac{adjacent}{hypotenuse} = \frac{V_x}{V}$$

$$V_x = Vcos\theta$$

So in finding the resultant of more than two vectors, we resolve each vector and then add up all their values (or

components) and then all their y values (or components). This would result into a simple drawing of x and y components. Thereafter we can find the magnitude and direction (or angle) of the diagonal which is the resultant.

For example: consider four forces acting on a body as shown below:



Resolving all the vectors to both x and y values:

$x_1 = F_1 cos\theta_1 \; ; x_2 = F_2 cos\theta_2 ; \; x_3 = F_3 cos\theta_3 \; ; \; x_4 = F_4 cos\theta_4 .$

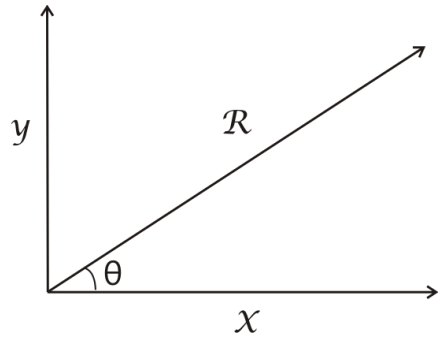
$y_1 = F_1 sin\theta_1 ; \; y_2 = F_2 sin\theta_2 ; \; y_3 = F_3 sin\theta_3 ; \; y_4 = F_4 sin\theta_4 .$

Adding up all the x values and all the y values:

$$x = x_1 + x_2 + x_3 + x_4$$

$$y = y_1 + y_2 + y_3 + y_4$$

Draw the sum x and y values and then find the values of the resultant (or diagonal) using Pythagoras rule.



$$R^2 = x^2 + y^2$$

$$R = \sqrt{x^2 + y^2}$$

To find the direction (or angle) of the resultant we simply use the appropriate trigonometric ratio.

$$tan\theta = \frac{x}{y}$$

$$\theta = \tan^{-1}\left(\frac{x}{y}\right)$$