# SIMPLE HARMONIC MOTION (SHM)

Motion is simply the change in position. Random motion, translatory motion, oscillatory motion, circular motion, and rotational motion are all types of motion.

When a body moves to and fro, reversing the direction of its motion and returning regularly to its original position, it is said to have a vibratory or oscillatory motion. A familiar example is the simple pendulum in which a small mass at the end of a string suspended from a point moves to and fro.

Simple Harmonic Motion is the periodic movement of a body or particle along a straight line such that the acceleration of the body is directed towards a fixed point (or centre of motion) and is also proportional to its displacement from that point.

From the definition above we find out that simple harmonic motion is just the same as oscillatory or vibratory motion.

In simple harmonic motion,

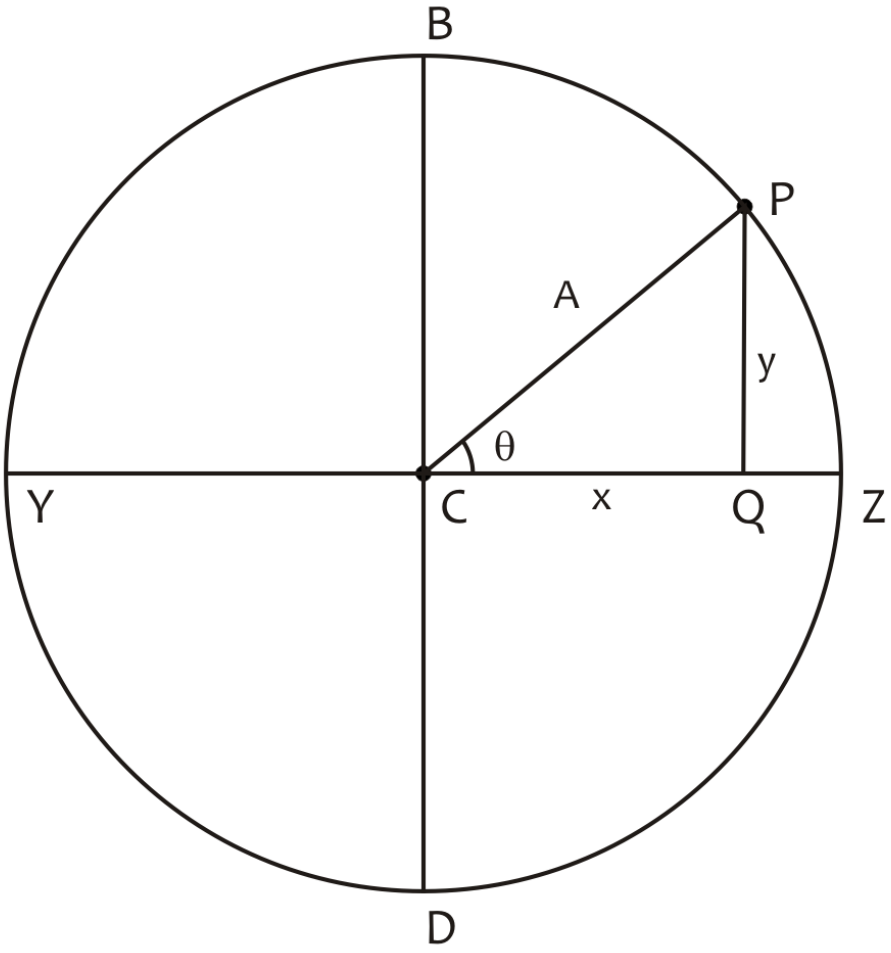
i. The acceleration of the vibrating body and the force acting to restore it to its equilibrium position (restoring force) are always proportional to the displacement of the body from the centre of its path
ii. The acceleration a, and the restoring force F, are in a direction opposite to that of the displacement (x), that is:

$$F \propto -x \quad or \quad F = -kx$$

where k is the force constant and the minus sign indicates the opposition between the force and displacement.

## Simple Harmonic Motion from Circular Motion

We can study SHM with a geometrical construction called Reference Circle. Consider a particle, P, moving round a reference circle with Centre C and diameter BD and YZ as shown below:




Let the particle move with a uniform speed, v round the circle of radius CZ = OY = A. The angular velocity $\omega$, of the radius vector CP is related to the speed v of the particle by the equation.

$$v = \omega A$$

Let us drop a perpendicular from P to the diameter YZ to meet YZ at Q. Q is called the projection of P upon the diameter YZ. As P moves round the circle, Q moves to and fro along YZ, being momentarily at rest at Y and Z and moving with maximum speed as it passes through C the centre of the path YZ. Thus, the motion of Q along the diameter YZ is simple harmonic motion while the particle P moves with a constant speed v round the circle.

From the diagram above, we can see that: $x = A\cos\theta$

where x is the horizontal distance of the particle P from the centre C at that point in time. As the particle P moves from Z through B, Y, D and back to Z, (i.e. round the circle) the angle $\theta$ swept through by the radius vector from Z to P varies form $0^o$ at Z to $90^o$ at B, to $180^o$ at Y, to $270^o$ at D and to $360^o$ or $0^o$ at Z. $\cos\theta$ varies from 1 to 0 to -1 to 0 and to 1 as x varies from the value A to 0 to -A to 0 and to A again. Hence, the value of x, The horizontal displacement from the central position

C, varies from the value A to -A or we can say $-A \leq x \leq A$.

The maximum value of x has the value A and is known as the amplitude of the motion. The amplitude of a simple harmonic motion is the maximum displacement of the body performing simple harmonic motion from its equilibrium or central position, C.

The total time taken by the point Q to move from Z to Y and back to Z is known as the Period, T. Period is defined as the total time taken by a vibrating body to make one complete revolution (or cycle) about a reference point.

If a body makes a given number of revolutions (n) in a given time, t, then the number of revolutions per unit time (t) is known as the frequency (f) of the SHM. Frequency is the number of complete revolution per second made by a vibrating body. Its SI unit is Hertz (Hz)

$$f = \frac{n}{t} = \frac{1}{T}$$

## Speed and Acceleration of SHM

In the reference circle, having a particle P moving round the circle, we notice that the particle P sweep through an angle of $\theta$ = 360° (or $2\pi$ radius) in the time T, the period of the motion. The rate of change of the angle $\theta$ with time (t) is known as the angular velocity $\omega$.

$$w = \frac{angle\ turned\ through\ by\ the\ body}{time\ taken} = \frac{\theta}{t}$$

For motion in a straight line

$$v = \frac{disatnce}{time\ taken} = \frac{s}{t}$$

The angle $\theta$ is measured in radians (rad) and since $2\pi$ rad = 360°, we have from $\omega = \frac{\theta}{t}$ that the angular velocity $\omega$ is measured in radians per seconds.

As the angle is changing with time so is the arc length s is changing with time. If $\theta$ is in rads, the length of the arc equals

$$s = r\theta$$

where r = A = radius of the circle. The angular velocity ($\omega$) is given by

$$w = \frac{\theta}{t} = \frac{s}{r}.\frac{l}{t} = \frac{s}{t} \times \frac{l}{r} = \frac{v}{r}$$

$$\Rightarrow v = \omega . r = \omega A$$

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