EQUATIONS OF UNIFORMLY ACCELERATED MOTION

There are equations for the motion of a body moving along a straight line with uniform acceleration. Below are the equations and their derivations:

The First Equation:

$$\boldsymbol{v = u + at}$$

Let the initial velocity of a body moving in a straight line be *u*, its acceleration be *a* and the final velocity be *v*. let the time of motion be *t*. Acceleration (*a*) is defined by:

$$a = \frac{change\ in\ velocity}{time\ interval} = \frac{v - u}{t}$$

Making *v* the subject of formula:

$$a = \frac{v - u}{t}$$

Multiply both sides by *t*,

$$at = \frac{(v - u)t}{t} = v - u$$

Add *u* to both sides,

$$v - u = at$$

$$v - u + u = at + u$$

$$v = u + at$$

The Second Equation:

$$\boldsymbol{s = ut + \frac{1}{2}at^2}$$

Let s be the distance covered in a time t, with the initial and final velocities given as *u* and *v* respectively.

$$average\ velocity\ (\bar{v}) = \frac{v + u}{2}$$

Making *v* the subject of formula,

$$\bar{v} = \frac{v + u}{2}$$

$$2\bar{v} = v + u$$

$$v = 2\bar{v} - u$$

Since from the first equation, $v = u + at$ and from above equation, $v = 2\bar{v} - u$, then we can equate both equations and then make $\bar{v}$ the subject of the formula

$$v = 2\bar{v} - u = u + at$$

$$2\bar{v} = u + u + at = 2u + at$$

$$\bar{v} = \frac{2u + at}{2} = \frac{2u}{2} + \frac{at}{2} = u + \frac{1}{2}at$$

Also, average velocity $\bar{v}$ is:

$$\bar{v} = \frac{distance}{time} = \frac{s}{t}$$

Since $\bar{v} = u + \frac{1}{2}at$ and is also equal to $\bar{v} = \frac{s}{t}$, we can equate both equations and make distance, s the subject of formula:

$$\bar{v} = \frac{s}{t} = u + \frac{1}{2}at$$

$$\frac{s}{t} = u + \frac{1}{2}at$$

$$s = \left(u + \frac{1}{2}at\right)t = ut + \frac{1}{2}at^2$$

$$s = ut + \frac{1}{2}at^2$$

The Third Equation:

$$\boldsymbol{v^2 = u^2 + 2as}$$

Method 1:

Using equations: $\bar{v} = \frac{u+v}{2}$, $\bar{v} = \frac{s}{t}$, and $t = \frac{v-u}{a}$

$$\bar{v} = \frac{u+v}{2} = \frac{s}{t}$$

$$s = \frac{u+v}{2} \times t$$

Since $t = \frac{v-u}{a}$, we put it into the above equation:

$$s = \left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right)$$

$$s = \frac{(v+u)(v-u)}{2a} = \frac{v^2 - uv + uv - u^2}{2a} = \frac{v^2 - u^2}{2a}$$

$$s = \frac{v^2 - u^2}{2a}$$

Making v^2 the subject of formula

$$s = \frac{v^2 - u^2}{2a}$$

$$v^2 - u^2 = 2as$$

$$v^2 = u^2 + 2as$$

Method 2:

Using equations: $v = u + at$ and $s = ut + \frac{1}{2}at^2$

Square both sides of $v = u + at$

$$v^2 = (u + at)^2 = u^2 + 2uat + a^2t^2$$

$$v^2 = u^2 + 2uat + a^2t^2 = u^2 + 2a\left(ut + \frac{1}{2}at^2\right)$$

Since $ut + \frac{1}{2}at^2 = s$, thus we substitute s:

$$v^2 = u^2 + 2a\left(ut + \frac{1}{2}at^2\right) = u^2 + 2as$$

$$v^2 = u^2 + 2as$$

Note: distance, s is also equal to $\left(\frac{v+u}{2}\right)t = \bar{v}t$