ADDITION OF PROBABILITIES

The probability of the occurrence of any two (or more) mutually exclusive event is equal to the sum of their separate probabilities. Thus, the probability of two events A and B which are mutually exclusive is the

$$P\ (A\ or\ B)\ =\ P(A)\ +\ P(B)$$

In set notation, "either A or B" can be written a '$A \cup B$'.

$$P\ (A \cup B) = P(A) + P(B)$$

The probability of events that are non-mutually exclusive, i.e. they overlap can be handled using:

$$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

$$\rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Examples

1. $A = \{a, e, i, o, u\}$ – vowels and $B = \{p, h, f, v, m, w\}$ – consonants. Find the probability of choosing a letter from the English alphabets which is either in event A or event B.

Solving

The total number of English alphabets = 26

A = 5; B = 6

$$P\ (A\ or\ B) = P\ (A) + P\ (B)$$
$$= \frac{5}{26} + \frac{6}{26} = \mathbf{\frac{11}{26}}$$

2. Find the probability of drawing from a pack of 52 well-shuffled card, (i) a spade (ii) a red card. Hence derive the probability of drawing either a spade or a red card. [Hint: spade = 13; red = 26]

Solving

The total cards = 52

(i) $P(spade) = \frac{13}{52} = \frac{1}{4}$

(ii) $P(red) = \frac{26}{52} = \frac{1}{2}$

(iii) $P(spade\ or\ red) = \frac{13+26}{52} = \frac{39}{52} = \frac{3}{4}$

3. A ball is drawn at random from a box containing 6 red balls, 4 white balls and 5 blue balls. Determine the probability that it is

a. red b. white c. blue

d. not red e. red or white

Solving

The total number of balls in the box = $6 + 4 + 5 = 15$

a. $P\ (red) = \frac{6}{15} = \frac{2}{5}$

b. $P(white) = \frac{4}{15}$

c. $P(blue) = \frac{5}{15} = \frac{1}{3}$

d. $P(red)' = 1 - P(red) = 1 - \frac{2}{5} = \frac{5}{5} - \frac{2}{5} = \frac{5-2}{5} = \frac{3}{5}$

e. $P(red\ or\ white) = P(red) + P(white) = \frac{2}{5} + \frac{4}{15} = \frac{6+4}{15} = \frac{10}{15} = \frac{2}{3}$