SURDS

Rational numbers are all numbers that can be expressed in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$. Rational numbers also include recurring or repeated decimals like $17\frac{1}{3} = 17.333333333\ldots$ written as $17.\dot{3}$. The dot above the 3 shows that only the digit 3 reoccurs or repeats indefinitely. If two or more digits have the dots on top of them, then those digits repeat in that order indefinitely, e.g. $17\frac{2}{11} = 17.18181818\ldots$ written as $17.\dot{1}\dot{8}$ with two digits repeating. $17\frac{7}{39} = 17.189189189\ldots$ written as $17.\dot{1}\dot{8}\dot{9}$.

All these recurring decimal numbers are rational because they can be expressed in the form $\frac{a}{b}$ with $b \neq 0$.

Example: Express in the form $\frac{a}{b}$ the number, $4.\dot{3}$

Solving

$4.\dot{3} = 4.3333$
$x = 4.3333$
$10x = 43.3333$
$10x - x = 43.3333 - 4.3333$
$9x = 39$
$x = \frac{39}{9} = \mathbf{\frac{13}{3}}$

But when we get non-repeating, or non-terminating decimal numbers, then we are dealing with irrational numbers. It is very difficult to deal with these non-repeating or non-ending decimal numbers in any calculations except perhaps now with the aid of modern technologies like the computers because they can continue almost indefinitely, e.g.

$$\frac{1}{17} = 0.058823529411764705882352941176\ldots$$

We define this kind of numbers as irrational or non-rational numbers which are numbers that cannot be expressed in the form $\frac{a}{b}$ with $b \neq 0$. The square root of all non-perfect squares are irrational numbers, e.g. $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \sqrt{8}, \sqrt{10}, \sqrt{11}, \sqrt{12}, \sqrt{13}\ldots$ Also all multiples of irrational

numbers are irrational numbers e.g. $3\sqrt{2}, 10\sqrt{3}, 6\sqrt{5}$, etc. and all fractions of irrational numbers e.g. $\frac{\sqrt{2}}{4}, \frac{3\sqrt{2}}{4}, \frac{\sqrt{3}}{5}$, etc.

The number π is really an irrational number even though for purposes of calculations we approximate it to its nearest rational number $\frac{22}{7}$.

Since irrational numbers have no ending, they do not have exact values so whenever we use them in calculations we use their approximate values. For example:

a. The square root tables give the approximate values of the square roots of non-perfect squares to four decimal places.
b. We have already stated for calculations we put $\pi = \frac{22}{7}\ or\ 3.142$.
c. We usually rationalise irrational numbers especially if found as denominators of some fractions to make more sense e.g. $\frac{3}{\sqrt{2}}$ can be rationalised as $\frac{3\sqrt{2}}{4}$. We call the square roots of non-perfect squares surds, e.g. $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}$, etc.

Rationalising the Denominator

Like we do for rational numbers, we often need to add, subtract, multiply and divide surds. These basic operations with surds would have posed great difficulties, especially diving by a surd, if we just use surds like that. So to simplify the process we adopt 2 methods:

a. By expressing a surd in terms of simpler surds if possible, e.g. $\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$ and
b. By rationalising the denominator. This is meant to make the denominator rational, and is done by multiplying both the numerator and the denominator by another surd or a "conjugate surd." For example, $\left(\sqrt{a} - \sqrt{b}\right)$ is the conjugate of $\left(\sqrt{a} + \sqrt{b}\right)$. We have that $\left(\sqrt{a} - \sqrt{b}\right)\left(\sqrt{a} + \sqrt{b}\right) = a + b$, which is a rational number. Thus using conjugate surds, sometimes we rationalise the original surds.