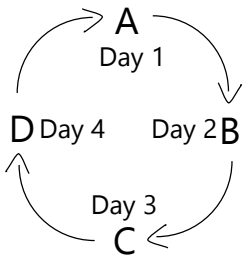
THE CONCEPT OF MODULAR ARITHMETIC

In many parts of the world, some activities like market days, clock timing, shift duty, menstrual cycle, anniversaries, etc. are carried out in rotation or cycle. These activities are held for every specific number of periods. For instance, some markets are held every four days or every week (i.e. every seven days).

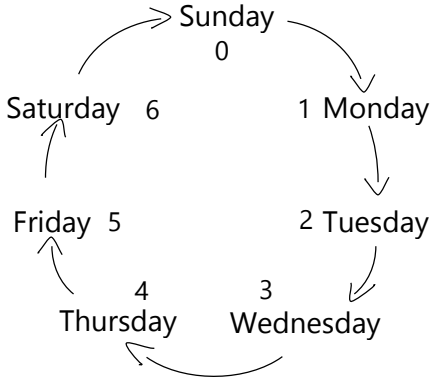
Now if for example, A, B, C, D, denote respectively the four market days Eke, Oye, Afor, and Nkwo in a place, then the fifth day will correspond to A where the market originally started. This information can be displayed in a cycle as shown below:



We can also use 0, 1, 2, 3 as codes for the markets instead of A, B, C, D respectively as shown below:

Market	A	B	C	D
No. Codes	0	1	2	3

Similarly, we can display the days of the week — Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, and Saturday on a circle using the number codes 0, 1, 2, 3, 4, 5, and 6 as shown below:



Consider the table below which shows some days of the 1st and 2nd weeks in a month when each market A, B, C, D, mentioned above was held:

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
1st week	A	B	C	D	A	B	C
2nd week	D	A	B	C	D	A	B
3rd week	C	D	A	B	C	D	A
4th week	B	C	D	A	B	C	D

From the table above, we can see that for instance market A was held on Sunday and Thursday of the first week. Market C was held only once in the second week.

We would also notice that markets C and A were held on the 1st and last days of the 3rd week respectively. Market B was held on Monday and Friday of the 1st week.

Now let's consider the same table but this time with the number code.

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
1st week	0	1	2	3	1	2	3
2nd week	3	0	1	2	3	1	2
3rd week	2	3	0	1	2	3	1
4th week	1	2	3	0	1	2	3

From the above table, we observe that 6 days after market A will correspond to market C. This is obtained by starting from market A denoted by 0 and counting round the cycle 6 points in a clockwise direction. This corresponds to 2 which is the denotation of C.

Using mathematical symbols, we express this as:

$$0 + 6 \rightarrow 2$$

Similarly 10 days after D (3) gives B (1):

$$3 + 10 \rightarrow 1$$

Since each market comes every 4 days, we can present the examples above as follows:

$0 + 6 = 6$
$6 \div 4 = 1\ rem\ 2$
$\therefore 0\ + 6 \rightarrow or\ \equiv 2$

$3\ + 10 = 13$
$13 \div 4 = 3\ rem\ 1$
$\therefore 3\ + 10 \rightarrow or\ \equiv 1$

Also from the table above we noticed that 5 days before market A corresponds to market D i.e. by starting from market A (denoted by 0) and counting round the cycle 5 points in an anticlockwise direction to arrive at D (denoted by 3).

In mathematical form this means

$0 - 5 \rightarrow 3$

But $0 - 5 = -5$

$-5 = -8 + 3$

$= -2 \times 4\ + 3$

$\therefore 0 - 5 \rightarrow 3$

From the above discussions, we have found a new notion of arithmetic for integers where numbers "wrap around" or "start over after reaching a certain value say *n* called the modulus or modulo. This type of arithmetic is called *modular arithmetic* (or sometimes called "clock arithmetic").

In our examples above, the modulus is 4 (since each market comes up every 4 days) so we write.

$6\ \equiv 2\ as$

$6 = 2\ (modulus\ 4)$

$or\ 6 = 2\ (modulo\ 4)$

$or\ 6 = 2\ (mod\ 4)$

Similarly

$10 = 2\ as\ (mod\ 4)$

$14 = 2\ as\ (mod\ 4)$

$2 = 2\ as\ (mod\ 4)$ since $2 = 0 \times 4 + 2\ etc.$

This means that 6, 10, 14, 2 have same remainder when divided by 4, i.e. 6, 10, 14, and 2 are equivalent with respect to mod 4 or equivalently that the difference between any two of 6, 10, 14, and 2 is divisible by 4 (or is a multiple of 4).

In general, if

$a = b\ (mod\ n)$ then *a* and *b* have the same remainder when divided by *n*, or equivalently that the difference between *a* and *b* is divisible by *n* (or a multiple of n).

Note that in modular arithmetic, the interest or emphasis is centred on the remainder after division.