NUMBER APPROXIMATION, ESTIMATION AND PERCENTAGE ERROR

Sometimes we do not need to measure or calculate things exactly. We may only wish to have a rough idea, or to calculate only to a certain degree of accuracy. It is in such cases, we talk of decimal places, significant figures or rounding off to the nearest unit, tens, etc. When this happens, we say that we are approximating.

Rounding off

In rounding off we recall the place value of numbers (e.g. 743,258) which we can write in the abacus form as.

HTH	TTH	TH	H	T	U
7	4	3	2	5	8

Now to round off this number to the nearest ten, for example, we look at the digit to the right 5 tens. If the digit is equal to, or greater than 5 (≥ 5) then we add 1 to the digit in the tens place. Like to the nearest ten, 743258 = 743260. But if the digit to the right of 5 tens is less than 5 (<5) then we do not add 1 to the tens value, i.e. to the nearest tens, 743253 = 743,250.

To round off a number to the nearest unit, usually involves numbers which are written in decimal form or mixed number form. If it's fractional part is greater than ½ we add 1 to the unit digit of the number. In the decimal form, if the digit after the decimal point is 5 or greater than 5, we add 1 to the unit digit of the number.

Decimal Places

In order to calculate to a required number of decimal place, we usually calculate to one place more than the required. If that last digit is less than 5, it would be discarded but if it is 5 or more than 5, then 1 is added to the digit just before it.

Examples: 45.478 to two decimal places (d.p.) =45.48; since 8 is more than 5 we add 1 to 7; 122.184 to two decimal places = 122.18, we discard 4 as it is less than 5.

We can express rounding off to a required number of decimal places in another way, thus: if to 1 decimal place, we can call it rounding off to the nearest tenth using our place value notation.

Significant Figures

The first non-zero digit in any number is its first significant figure. A zero between two significant figures is counted as a significant figure. We calculate to a required number of significant places by the same method as we do for decimal places. Numbers can be rounded off to any required number of significant figures.

Examples

i. To 3 significant figures, 0.02364 = 0.0236
ii. To 2 significant figures, 5286 = 5200
iii. To 1 significant figure, 9.103 = 9

Degree of Accuracy of Results

When we round off a number to the nearest 1 or 10 or 100, etc. we are not considering the exact value and also many other numbers can round off to that round number, e.g. to the nearest ten, 25 = 30, 26 = 30, 27 = 30, 28 = 30, 29 = 30, 30 = 30, 31 = 30, 32 = 30, 33 = 30, 34 = 30. But not 35 which rounds off to the nearest ten, as equal to 30. So there is certainly some admitted error rounding off. So that the nearest ten, 30 can be the number *N*, between 25 and 34 but not 35, or in words, *N* lies between 25 and 35 written mathematically as:

$$25 \leq N < 30 \; or \; N = 30 \pm 5 \; or \; [N = 29.5 \pm 4.5]$$

This is the limit of the accuracy of our results. 25 is the lower limit and 35 is the upper limit of the range. We find this limit by halving the range. We find this limit by halving the range of accuracy and adding and subtracting from the rounded number.

NOTE: the above somehow relates to histogram's upper and lower boundaries.

Estimation

Estimation is a mental work which uses rounding off process in order to have an idea of a result. It acts as a check, or a guide to the correct result.

Example: Estimate the value of 483 × 67 and then compare it with the actual answer.

Solving

Round off 483 to the nearest 100 = 500
Round off 67 to the nearest 10 = 70
Rough estimate = 500 × 70 =35000

Actual multiplication = 483 × 67 = 32,361

We can see that the actual product is a value near to the estimated product.

Percentage Error

When we make estimate or approximations, we do not have the exact value of the result, but an approximation to it, i.e. an idea of the value.

Now the difference between the actual result and the estimated or approximated result can be calculated in percentage. This is known as the percentage error.

Note that the nearer the percentage error is to zero, the more accurate the result.

Example: The exact distance a boy walks to school every morning is 4km. When asked, he said 4½km while his parents said he walks 3km to school. Find their percentage errors.

Solving

Percentage error of the boy

$$\frac{4\frac{1}{2}-4}{4}\times 100=\frac{\frac{1}{2}}{4}\times 100=\frac{1}{8}\times 100=12.5\%$$

Percentage error of the parents

$$\frac{4-3}{4} \times 100 = \frac{1}{4} \times 100 = 25\%$$

So the boy is more accurate than the parents.

The Absolute Error is the difference between the estimate and the exact values. This percentage error is usually referred to as the Relative Error, i.e. Relative Error is often given as a percentage.

$$Relative\ or\ Percentage\ Error = \frac{absolute\ error}{exact\ value} \times 100$$