SIMULTANEOUS EQUATIONS AND SOLVING THEM

The term 'simultaneous' implies that two things occur together. Therefore, simultaneous equation imply equations that occur together. In other words, they relate in that they share variable or variables.

For example:

$$10x + 46 = y$$

$$y - 13 = 12x$$

The both equations above are simultaneous as they share the variables x and y. The simultaneous equations may be linear, quadratic, cubic or any other kind of equations. For example:

$$10x + 46 = y$$

$$y = x^2 + x - 3$$

Simultaneous Linear Equations

Simultaneous Linear Equations can be solved basically by any of these three methods:

- Substitution method
- Elimination method
- Graphical method

Substitution Method

This method involves reducing one of the equations such that one of the variables would become the subject of the equation. Thereafter the resultant expression is substituted into the value of that particular variable such that the new equation would have only one variable (or rather a function of only one variable) and can then be simplified by simple means.

Example: simplify $3x + y = 1$ and $2x - 3y = 8$

Solving

$3x + y = 1$ ---- *(i)*

$2x - 3y = 8$ ---- *(ii)*

$2x = 8 + 3y$

$x = \frac{8+3y}{2}$ ---- *(iii)*

Put the value of x (eqn. *iii*) into equation *ii*.

$3x + y = 1$

$3\left(\frac{8+3y}{2}\right) + y = 1$

$\frac{24+9y}{2} + y = 1$

$\frac{24+9y}{2} + \frac{2y}{2} = 1$

$\frac{24+9y+2y}{2} = 1$

$24 + 9y + 2y = 2$

$24 + 11y = 2$

$11y = 2 - 24 = -22$

$y = -\frac{22}{11} = -2$

$\boldsymbol{y = -2}$

Now we've found the value of y as *-2*. To find the value of x we can simply put the value of y into any of the above equation, whether *(i)*, *(ii)* or *(iii)*.

$3x + y = 1$

$3x + (-2) = 1$

$3x - 2 = 1$

$3x = 1 + 2 = 3$

$x = \frac{3}{3} = 1$

$\boldsymbol{x = 1}$

$$\therefore \boldsymbol{x = 1}\ and\ \boldsymbol{y = -2}$$

Elimination Method

This method can be used when one of the variable has the same coefficient in the equations involved. Most of the times the coefficients may not be the same, so to make them the same we use numbers that can multiply through the equations to make the coefficients of their variables to be the same. Thereafter, you subtract one of the equations from the other.

Example: simplify $3x + y = 1$ and $2x - 3y = 8$

Solving

$3x + y = 1$ --- (i)
$2x - 3y = 8$ --- (ii)

Equations *i* and *ii* do not share same coefficients for either x or y. Since we wish to eliminate x and its coefficients so we balance its coefficients of *x* by multiplying the equations *i* and *ii* with 2 and 3 respectively.

$2(3x + y = 1) = 6x + 2y = 2$ --- (iii)
$3(2x - 3y = 8) = 6x - 9y = 24$ --- (iv)

Now the coefficients of x in both equations are the same, so we can eliminate or subtract any of the equations, *iii* and *iv* from each other.

(iii) – (iv)

$$\begin{array}{rl} & 6x + 2y \quad = 2 \\ - & (6x - 9y \quad = 24) \\ \hline & 0x + 11y = \text{-}22 \end{array} \quad \text{--- (v)}$$

Equation *v* is simply,

$11y = -22$
$y = \frac{-22}{11} = -2$

$\boldsymbol{y = 2}$

As we've gotten the value for y, we then insert it into any of the first equation, *i, ii, iii* or *iv*. Let's use *i*.

$3x + y = 1$
$3x + (-2) = 1$
$3x - 2 = 1$
$3x = 1 + 2 = 3$
$x = \frac{3}{3} = 1$

$\boldsymbol{x = 1}$

$$\therefore \boldsymbol{x = 1} \; and \; \boldsymbol{y = -2}$$