FORMULATION OF QUADRATIC EQUATIONS FROM GIVEN ROOTS

An equation can be formed if its roots are given. So given that the roots of a quadratic equation are α and β then

Either $x = \alpha \; or \; x = \beta$

i.e. $x - \alpha = 0 \; or \; x - \beta = 0$

$(x - \alpha)(x - \beta) = 0$

$x^2 - \alpha x - \beta x + \alpha\beta = 0$

$x^2 - x(\alpha + \beta) + \alpha\beta = 0$

The required equation which has the

1. The coefficient of x^2 is unity
2. The sum of the root, $\alpha + \beta$ is the coefficient of x with the sign changed
3. The product of the roots $\alpha\beta$ is the constant term

Thus, if the roots of a quadratic equation are given, then the equation can be obtained (with the usual three terms) as:

$$x^2 - (sum \; or \; roots)x + (product \; of \; roots) = 0$$

In comparsion to the normal quadratic equation:

$$x^2 - x(\alpha + \beta) + \alpha\beta = ax^2 + bx + c$$

$$x^2 - x(\alpha + \beta) + \alpha\beta = x^2 + \frac{b}{a}x + \frac{c}{a}$$

$$\alpha + \beta = -\frac{b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

$$\therefore sum \; of \; roots = -\frac{b}{a} \; and \; product \; of \; roots = \frac{c}{a}$$

Example: find the quadratic equation whose roots is 5 and 6

Solving

$x^2 - (sum\ of\ roots)x + product\ of\ roots = 0$

$x^2 - (5 + 6)x + (6 \times 5) = 0$

$x^2 - 11x + 30 = 0$

Example: find the sum and product of the roots of the quadratic equation, $2x^2 + 3x - 1 = 0$.

Solving

$2x^2 + 3x - 1 = 0$

$a = 2; b = 3; c = -1$

$Sum\ of\ roots = -\frac{b}{a} = -\frac{3}{2}$

$Product\ of\ roots = \frac{c}{a} = -\frac{1}{2}$