VARIATION: DIRECT VARIATION

In everyday life, we come across relationship between quantities. These relationships may be such that as one increases the other reduces or even, vice versa. This relationship can be called *Variation*.

The variations of quantities can be in many forms depending on the quantities concerned: (i) direct variation, (ii) indirect variation, (iii) joint variation, and (iv) partial variation.

Direct Variation

Now consider two variables, *x* and *y*. If when *y* increases, *x* increases also; or when *y* decreases, *x* decreases in the same proportion, then we say that the quantities *y* and *x* are in direct variation, or that are in direct variation, or that *y* varies directly as *x*, or that *y* is directly proportional to *x*. This is written as $y \propto x$ where $\propto$ is the *sign of variation* or *proportionality*.

For instance, 2 books cost ₦100 and 4 books cost ₦200. Therefore the increase in number of books causes an increase in price. Therefore, the number of books is directly proportional to cost, $b \propto c$.

$$\frac{c}{b} = \frac{100}{2} = \frac{200}{4} = \frac{5}{1}$$

Since in the first instance, the amount of books where 2 and the price was N100, when divided we get $\frac{5}{1}$. In the second instance, after dividing we also had $\frac{5}{1}$. In essence that $\frac{5}{1}$ is the constant.

$$\therefore \frac{c}{b} = constant\ (k)$$

$$c \propto b \rightarrow c = constant\ (k) \times b$$

$$Constant, k = \frac{c}{b}$$

Example: the number n of bottles of wine is directly proportional to the cost C of the bottles of wine. If 10 bottles of wine cost ₦400.

a. What is the cost of 18 bottles?

b. How many bottles can ₦200 buy?

Solving

$n \propto c \ \rightarrow n = kc \rightarrow k = \frac{n}{c}$

$n = 10; c = 400$

$\therefore k = \frac{10}{400} = \frac{1}{40}$

a. $n = 18; k = \frac{1}{40}; c = ?$

$k = \frac{n}{c} \rightarrow c = \frac{n}{k} = 18 \div \frac{1}{40} = 18 \times 40 = 720$

$c = 720$

The cost of 18 bottles is ₦720.

b. $n = ?; k = \frac{1}{40}; c = 200.$

$k = \frac{n}{c} \rightarrow n = kc$

$n = \frac{1}{40} \times 200 = 5.$